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# Eight Mathematical Biographies

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## Contents

## Introduction

When a *Dictionary of Scientific Biography* was planned, my special research interest was Giuseppe Peano, so I volunteered to write five entries on Peano and his friends/colleagues, whose work I was investigating. (The *DSB* was published in 14 volumes in 1970–76, edited by C. C. Gillispie, New York: Charles Scribner's Sons.) I was later asked to write two more entries: for Parseval and Emil Leon Post. The entry for Post had to be done very quickly, and I could not have finished it without the generous help of one of his relatives. By the time the last of these articles was published in 1976, that for Giovanni Vailati, I had come out publicly as a homosexual and was involved in the gay liberation movement. But my article on Vailati was still discreet. If I had written it later, I would probably have included evidence of his homosexuality.

The seven articles for the *Dictionary of Scientific Biography* have a uniform appearance. (The exception is the article on Burali-Forti, which I present here as I originally wrote it—with reference footnotes. As published, these references were simply compressed into a bibliography, which was the uniform format for the *DSB*.) I have also included an article on Maria Gaetana Agnesi, which I was asked to write much later for the volume *Women of Mathematics* (1987). The format of the article is similar to the others and it seems appropriate to include it here. Agnesi had interested me ever since I had acquired a copy of her two-volume textbook *Instituzioni Analitiche* (1748). It is a beautiful publication and I was proud to have it, but when I moved from Providence, Rhode Island, to San Francisco in 1986, I gave the two volumes to friends in Italy, a lesbian couple, both professors of mathematics with an interest in the history of mathematics.

Portraits of four of these mathematicians may add interest to this collection.

Maria Gaetana Agnesi (1718–1799)

WOMEN OF MATHEMATICS
A Biobibliographic Sourcebook
Edited by Louise S. Grinstein and Paul J. Campbell
With a Foreword by Alice Schafer
Greenwood Press, 1987
New York • Westport, Connecticut • London
Pp. 1–5

# MARIA GAETANA AGNESI (1718–1799)

Hubert Kennedy

## BIOGRAPHY

Maria Gaetana Agnesi was born on May 16, 1718, the first of the twenty-one children of her father, Pietro Agnesi. Her mother, Anna Fortunata (Brivio) Agnesi, died on March 13, 1732, after giving birth to her eighth child; Pietro Agnesi later remarried. Both of Agnesi's parents were of wealthy merchant families of Milan, so that Pietro Agnesi could afford to lead the life of a cultured nobleman; indeed, he eventually purchased a title.

Agnesi's intellectual ability—particularly her excellent memory—was discovered early. She was trained in several languages and first exhibited by her father at the age of nine at one of his "academic evenings," where she recited, in Latin, a scholastic exercise on the topic: The study of the liberal arts by women is by no means to be rejected. Having discovered that his second daughter, Maria Teresa (1720–1795), had musical talent, Pietro Agnesi had her specially trained also; and his academic evenings, at which Maria Agnesi debated learned guests on philosophic and scientific topics, and her sister entertained at the harpsichord, became famous in Milan. These evenings continued until 1739, when Agnesi expressed a wish to retire to a convent of nuns. Her father opposed this; always obedient to him, she agreed not to do so. In return, he agreed to three wishes: (1) that she be allowed to dress simply and modestly, (2) that she might go to church whenever she wished, and (3) that she not be required to attend balls, the theater, and so on.

There followed a decade in which Agnesi concentrated her studies on mathematics. As early as 1735, she had corresponded with her teacher Carlo Belloni about a difficulty in Guillaume François de L'Hôpital's treatise on conic sections; she also had two teachers of mathematics and science: Francesco Manara (Pavia) and Michele Casati (Turin), both of whom later became university professors. From 1740 she was directed in her studies by Ramiro Rampinelli, professor of mathematics at the University of Pavia. This more systematic study included Charles René Reyneau's *Analyse démontré* (1708), which was an early attempt to bring order to the new mathematical discoveries of the seventeenth century. The result of her study was the publication in 1748 of the *Instituzioni Analitiche* (Foundations of Analysis), which was modeled on Reyneau's book and is the work on which Agnesi's fame as a mathematician rests.

The *Instituzioni Analitiche* is a systematic presentation, in two volumes, of algebra, analytic geometry, calculus, and differential equations. Already before its publication she had been elected, on Rampinelli's recommendation, a member of the Accademia delle Scienze (Bologna); the book made her widely known, as evidenced by a scene from Carlo Goldoni's play *Il medico olandese* (1756), in which the maid Carolina says to Monsieur Guden:

> You wonder that my mistress likes the sweet study of geometry? You should rather be astonished that you don't know that a woman has produced such a profound and great book. Its author is Italian, not Dutch, an illustrious and learned lady, an honor to her country.

Agnesi dedicated her book to the Empress Maria Theresa of Austria (to whom Agnesi's musical sister had earlier dedicated a volume of songs), and in return she received a crystal box with diamonds and a diamond ring. But the pious Agnesi was probably most pleased by the response of Pope Benedict XIV, who not only sent her a gold wreath set with precious stones and a gold medal, but also named her honorary professor at the University of Bologna; the diploma to this effect is dated October 5, 1750.

This was the period of the Enlightenment, and both Maria Theresa and Benedict XIV were relatively enlightened monarchs. Indeed, Italy had a long tradition of women professors. Nevertheless, Agnesi never lectured in Bologna, despite being urged to do so by, among others, the physicist Laura Bassi Verati (1711–1778), who had taught there since 1732 and whose first philosophical disputation had been heard by Benedict XIV while he was archbishop of Bologna.

Following the death of her father on March 19, 1752, a new phase of Agnesi's life began that lasted until her death. She restricted her study to theology and gave her time, effort, and money to devotional and charitable activities. Although continuing to live with her family, she kept a separate apartment, where she cared for a few poor, sick people. From 1759 she lived in a rented house with four of her poor people; and when money was needed for her charitable activity, she sold her gifts from the Empress Maria Theresa to a rich Englishman. Besides caring for the sick and indigent, she often taught catechism to working-class people.

In 1771 Prince Antonio Tolemeo Trivulzio gave his palace to be a home for the aged, named Pio Albergo Trivulzio. At the request of Cardinal Pozzobonelli, Agnesi assumed the position of visitor and director of women. The home was soon expanded to house 450 people. Agnesi herself moved there permanently in 1783 and devoted her last twenty-eight years to this institution.

The early years of intense study appear to have affected Agnesi's health as an adolescent, for her doctor at the time prescribed more physical exercise, including dancing and horseback riding, and these were followed by seizures of chorea, or St. Vitus's dance. But despite her delicate health and the voluntary privations of her later life, she continued to be physically active, although there was a decline in her last years, when she gradually grew blind and deaf. Fainting spells were followed by attacks of dropsy of the chest (hydrothorax); and the latter appears to have been the immediate cause of her death on January 9, 1799. She was buried in a common grave of poor people.

## WORK

Agnesi not only wrote but also supervised the printing of the *Instituzioni Analitiche*. It was completed near the end of 1748 on the presses of the publishing house Richini, which had been installed in her house! The typesetters gave credit for their later good work to the experience gained there. The book is in two quarto volumes of 1,020 pages, with an additional 59 pages of figures engraved by Marc'Antonio Dal Rè, which may be folded out to view while reading the text. The pages are of handmade paper and printed in large type with wide margins.

The two volumes contain the analysis of finite and infinitesimal quantities, respectively. Although Agnesi made no claim to original mathematical discoveries, nevertheless she revised the material considerably in an attempt to put into their "natural order" the discoveries that are "separated, without order, and scattered here and there in the works of many authors, and principally in the *Acta* of Leipzig, in the *Mémoires* of the Academy of Paris, and in other journals. . . . Then in the act of handling the various methods, there occurred to me several extensions and a number of other things, which by chance are not without novelty and originality." With a charming frankness, she also notes that she at first intended to publish the work in Latin; but having written it in Italian, she decided to avoid the effort of translation. Nor did she lay claim to purity of language, "having had in mind more than anything else the necessary possible clarity." It is just this clarity that has been praised by later commentators.

As an illustration of how contemporary mathematicians viewed this work, we cite the conclusion of the report of Jean d'Ortous de Mairan and Étienne Mignot de Montigny, who reviewed it for the Académic des Sciences (Paris):

> Obviously it covers all the analysis of Descartes and almost all of the discoveries which have been made up to the present in the differential and integral calculus. It takes a good deal of knowledge and skill to reduce to an almost always uniform method, as indeed was done, the various discoveries in the works of modern geometers, where these are often explained by methods quite different one from another. Order, clarity, and precision reign in every part of this work. Up to now we have seen no work, in any language, which allows the student to penetrate so quickly and so far into mathematical analysis. We regard this treatise as the most complete and well written of its kind. (Anzoletti 1900, 479–480)

That the work continued to be useful is shown by its later translation into French (second volume only) and English. It was widely used in Italy. Joseph Louis Lagrange listed it among the books he thoroughly studied, after Euclid's *Elements* and Alexis Claude Clairaut's *Algébre*, and before he read Leonhard Euler and Johann Bernoulli. It was soon overshadowed, however, by the series of systematic texts published by Euler, beginning with his *Introductio in Analysin Infinitorum* in 1748.

The name of Agnesi is most often recalled today in connection with the Curve of Agnesi, known in English texts as the Witch of Agnesi. This last term appears to be John Colson's mistranslation of *versiera*, the Italian form of the Latin name *versoria*, both used for this curve by Guido Grandi as early as 1718. The curve itself had already been described by Pierre de Fermat and Isaac Newton. Although the metric properties of the Curve of Agnesi continued to interest mathematicians, physical applications have been found only recently.

Agnesi first presented the *versiera* as an exercise in analytic geometry (*Instituzioni Anatitiche*, 380–382), where the problem is to find the equation of a curve described geometrically. This equation is usually given today as $y(a^2 + x^2) = a^3$, which is the inverse of the function given by Agnesi; but its graph (Table 35, Figure 162) looks the same, since she considers the $x$-axis to be the vertical axis and the $y$-axis to be the horizontal axis. She later presented an algebraic method for finding the curve's point of inflection (pp. 427–428), returning again to the curve to illustrate the method of derivatives for finding points of inflection (pp. 561–562).

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Works about Maria Gaetana Agnesi

Anzoletti, Luisa. *Maria Gaetana Agnesi*. Milan: L. F. Cogliati, 1900. In Italian.

Corrects and completes Frisi's essay of a century earlier. Includes genealogical table.

Frisi, Antonio Francesco. *Elogio storico di Donna Maria Gaetana Agnesi Milanese, dell'Accademia dell'Instituto delle Scienie, e Lettrice Onoraria di Matematiche nella Università di Bologna.* Milan: Galeazzi, 1799. Reprint. Milan, 1965. French translation by Antoine Marie Henri Boulard. Paris, 1807.

Published four months after Agnesi's death by a family friend, this work is the basis for Anzoletti's more extensive biography.

Kennedy, Hubert. "The witch of Agnesi—exorcised." *Mathematics Teacher* 62 (1969): 480–482.

Gives Agnesi's presentation of the *versiera* and calls attention to two widely repeated errors about Agnesi: that she and/or her father taught at the University of Bologna and that she became a nun.

Kramer, Edna A. "Maria Gaetana Agnesi." In *Dictionary of Scientific Biography*, edited by C. C. Gillispie. Vol. 1, 75–77. New York: Charles Scribner's Sons, 1970.

Mistakenly reports that Agnesi's father was a professor at the University of Bologna, but is otherwise good.

Loria, Gino. *Curve piane speciali algebriche e transcendenti: Teoria e storia*. 2 vols. Milan: 1900.

See pp. 93–99 for the Curve of Agnesi.

Masotti, Amaldo. "Maria Gaetana Agnesi." *Rendiconti del seminario matematico e fisico di Milano* 14 (1940): 89–127.

Updates Anzoletti's biography, without finding errors in it. The appendix (pp. 122–127) has an annotated list of the twenty-five volumes of manuscripts of Agnesi in the Biblioteca Ambrosiana (Milan).

Mulcrone, T. F. "The names of the curve of Agnesi." *American Mathematical Monthly* 64 (1957): 359–361.

Rebière, A. *Les femmes dans la science*, 2nd ed. Paris: Nony, 1897.

Mistakenly says that Agnesi joined an order of nuns, but is otherwise good.

Spencer, Roy C. “Properties of the witch of Agnesi—application to fitting the shapes of spectral lines.” *Journal of the Optical Society of America* 30 (1940): 415–419.

States that the curve is “of importance to physicists because it approximates the spectral energy distribution of x-ray lines and optical lines, as well as the power dissipated in sharply tuned resonant circuits” (p. 416).

Tenca, Luigi. “La versiera di . . . Guido Grandi.” *Bollettino dell’Unione Matematica Italiana* (3) 12 (1957): 458–460.

Thomas à Kempis, Sister Mary. “The walking polyglot.” *Scripta Mathematica* 6 (1939): 211–217.

Pleasant hagiography; but despite a reference to Anzoletti’s work, it apparently was not read, for the error that Agnesi and her father taught at the University of Bologna is again repeated.

On p. 288:

HUBERT KENNEDY is the editor/translator of Selected Works of Giuseppe Peano (1973), author of *Peano: Life and Works of Giuseppe Peano* (1980; Italian translation 1983), and translator of the novel *The Hustler* (1984) from the German original of John Henry Mackay. Currently a consulting editor of the *Journal of Homosexuality*, he is preparing a biography of Karl Heinrich Ulrichs (1825–1895), pioneer of the modern gay movement.

*Dictionary of Scientific Biography,* Vol. 2 (1970), Charles C. Gillispie, ed., New York: Charles Scribner's Sons, 1970, pp. 593–594.

BURALI-FORTI, CESARE (b. Arezzo, Italy, 13 August 1861; d. Turin, Italy, 21 January 1931); *mathematics*.

After obtaining his degree from the University of Pisa in December 1884, Burali-Forti taught at the Scuola Tecnica in Augusta, Sicily. In 1887 he moved to Turin after winning a competition for extraordinary professor at the Accademia Militare di Artiglieria e Genio. In Turin he also taught at the Scuola Tecnica Sommeiller until 1914. He remained at the Accademia Militare, teaching analytical projective geometry,[1] until his death. He was named ordinary professor in 1906 and held a prominent position on the faculty; in 1927 he was the only ordinary among twenty-five civilian professors.

After an early attempt to obtain the *libera docenza* failed because of the antagonism to the new methods of vector analysis on the part of some members of the examining committee, he never again attempted to obtain it and thus never held a permanent university position. (The *libera docenza* gave official permission to teach at a university and was required before entering a competition for a university chair.) He was assistant to Giuseppe Peano at the University of Turin during the years 1894–1896, but he had come under Peano's influence earlier, however, and had given a series of informal lectures at the university on mathematical logic (1893–1894). These were published in 1894.[2] Many of Burali-Forti's publications were highly polemical, but to his family and his friends he was kind and gentle. He loved music; Bach and Beethoven were his favorite composers. He was not a member of any academy. Always an independent thinker, he asked that he not be given a religious funeral.

The name Burali-Forti has remained famous for the antinomy he discovered in 1897 in his critique of Georg Cantor's theory of transfinite ordinal numbers.[3] The critique begins: "The principal purpose of this note is to show that there exist *transfinite numbers* (or *ordinal types*) *a*, *b*, such that *a* is neither equal to, nor less than, nor greater than *b*." Essentially, the antinomy may be formulated as follows: To every class of ordinal num-

1. *Lezioni di geometria metrico-proiettiva* (Turin, 1904).
2. *Logica matematica* (Milan, 1894; 2nd ed., rev., Milan, 1919).
3. "Una questione sui numeri transfiniti," in *Rendiconti del Circolo matematico di Palermo*, 11 (1897), 154–164.

bers there corresponds an ordinal number which is greater than any element of the class. Consider the class of all ordinal numbers. It follows that it possesses an ordinal number that is greater than every ordinal number. This result went almost unnoticed until Bertrand Russell published a similar antinomy in 1903. It should be noted, however, that Cantor was already aware of the Burali-Forti antinomy in 1895 and had written of it to David Hilbert in 1896.

Burali-Forti was one of the earliest popularizers of Peano's discoveries in mathematical logic. In 1919 he published a greatly enlarged edition of the *Logica mathematica*, which contained many original contributions. He also contributed much to Peano's famous *Formulaire de mathématiques* project, especially with his study of the foundations of mathematics (1893).[4]

Burali-Forti's most valuable mathematical contributions were his studies devoted to the foundations of vector analysis and to linear transformations and their various applications, especially in differential geometry. A long collaboration with Roberto Marcolongo was very productive. They published a series of articles in the *Rendiconti del Circolo matematico di Palermo* on the unification of vectorial notation that included a full analysis, along critical and historical lines, of all the notations that had been proposed for a minimal system. There followed a book treating the fundamentals of vector analysis (1909),[5] which was almost immediately translated into French.[6] Their proposals for a unified system of vectorial notation, published in *L'enseignement mathématique* in 1909,[7] gave rise to a polemic with various followers of Josiah Gibbs and Sir William Hamilton that lasted into the following year and consisted of letters, responses, and opinions contributed by Burali-Forti and Marcolongo, Peano, G. Comberiac, H. C. F. Timerding, Felix Klein, E. B. Wilson, C. G. Knott, Alexander Macfarlane, E. Carvallo, and E. Jahnke. The differences in notation continued, however, and the Italian school, while quite productive, tended to remain somewhat isolated from developments elsewhere. Also in 1909 Burali-

4. *Teoria delle grandezze* (Turin, 1893).

5. *Elementi di calcolo vettoriale, con numerose applicazioni alla geometria, alla meccanica e alla fisica-matematica*, written with R. Marcolongo (Turin, 1909).

6. *Eléments de calcul vectoriel, avec de nombreuses applications à la géométrie, à la mécanique et à la physique mathématique*, translated by S. Lattès (Paris, 1910).

7. "Notations rationelles pour le système vectoriel minimum," in *L'enseignement mathématique*, 11 (1909), 41–45, written with Marcolongo.

Forti and Marcolongo began their collaboration in the study of linear transformations of vectors.[8]

Burali-Forti's introduction of the notion of the derivative of a vector with respect to a point allowed him to unify and greatly simplify the foundations of vector analysis. The use of one simple linear operator led to new applications of the theory of vector analysis, as well as to improved treatment of operators previously introduced, such as Lorentz transformations, gradients, and rotors, and resulted in the publication (1912–1913) of two volumes treating linear transformations and their applications.[9] Burali-Forti was able to apply the theory to the mechanics of continuous bodies, optics, hydrodynamics, statics, and various problems of mechanics, always refining methods, simplifying proofs, and discovering new and useful properties. He did not live to see the completion of his dream, a small encyclopedia of vector analysis and its applications. The part dealing with differential projective geometry (1930) was Burali-Forti's last work.[10]

The long collaboration with Marcolongo—their friends called them "the vectorial binomial"—was partly broken by their divergent views on the theory of relativity, the importance of which Burali-Forti never understood. With Tommaso Boggio he published a critique (1924)[11] in which he meant "to consider Relativity under its mathematical aspect, wishing to point out how arbitrary and irrational are its foundations." "We wish," he wrote in the preface, "to shake Relativity in all its apparent foundations, and we have reason for hoping that we have succeeded in doing it." At the end he stated: "Here then is our conclusion. Philosophy may be able to justify the space-time of Relativity, but mathematics, experimental science, and common sense can justify it NOT AT ALL."

Burali-Forti had a strong dislike for coordinates. In 1929, in the second edition of the *Analisi vettoriale generale,* written with Marcolongo, we find: "The criteria of this work . . . are not different from those with which we began our study in 1909, namely, an absolute treatment of all physical, mechanical, and geometrical problems, independent of any system of coordinates whatsoever."

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8. *Omografie vettoriali con applicazioni alle derivate rispetto ad un punto ed alla fisica-matematica* (Turin, 1909), written with Marcolongo.

9. Analyse vectorielle générale, 2 vols. (Pavia, 1912–1913), written with Marcolongo.

10. *Analisi vettoriale generale e applicazioni.* Vol. II, *Geometria differenziale* (Bologna, 1930), written with P. Burgatti and Tommaso Boggio.

11. *Espaces courbes. Critique de la relativité* (Turin, 1924), written with Tommaso Boggio.

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Besides his scientific publications, Burali-Forti wrote many school texts. In all, his publications total more than two hundred.

No complete list of the works of Burali-Forti has been published, but the works listed in the footnotes may be considered representative.

A work dealing with Burali-Forti is Roberto Marcolongo, “Cesare Burali-Forti,” in *Bollettino dell’Unione matematica italiana*, 10 (1931), 182–185.

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*Dictionary of Scientific Biography*, ed. C. C. Gillispie. New York: Charles Scribner's Sons. 10 (1974): 274.

PADOA, ALESSANDRO (b. Venice, Italy, 14 October 1868: d. Genoa, Italy, 25 November 1937), *mathematical logic, mathematics*.

Padoa attended a secondary school in Venice, the engineering school in Padua, and the University in Turin, from which he received a degree in mathematics in 1895. He taught in secondary schools at Pinerolo, Rome, and Cagliari, and (from 1909) at the Technical Institute in Genoa.

Padoa was the first to devise a method for proving that a primitive term of a theory cannot be defined within the system by the remaining primitive terms. This method was presented in his lectures at Rome early in 1900 and was made public at the International Congress of Philosophy held at Paris later that year. He defined a system of undefined symbols as irreducible with respect to the system of unproved propositions when no symbolic definition of any undefined symbol can be deduced from the system of unproved propositions. He also said:

> To prove that the system of undefined symbols is irreducible with respect to the system of unproved propositions, it is necessary and sufficient to find, for each undefined symbol, an interpretation of the system of undefined symbols that verifies the system of unproved propositions and that continues to do so if we suitably change the meaning of only the symbol considered ["Essai . . .," p. 322].

Although it took the development of model theory to bring out the importance of this method in the theory of definition, Padoa was already convinced of its significance. (A proof of Padoa's method was given by Alfred Tarski in 1926 and, independently, by J. C. C. McKinsey in 1935.)

In lectures at the universities of Brussels, Pavia, Bern, Padua, Cagliari, and Geneva, Padoa was an effective popularizer of the mathematical logic developed by Giuseppe Peano's "school," of which Padoa was a prominent member. He was also active in the organization of secondary school teachers of mathematics and participated in many congresses of philosophy and mathematics. In 1934 he was awarded the ministerial prize in mathematics by the Accademia dei Lincei.

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Padoa's method was stated in "Essai d'une théorie algébrique des nombres entiers, précéde d'une introduction logiquc à une théorie déductive quelconque," in *Bibliothéque du Congrès international de philosophie*, Paris, 1900, III (Paris, 1901), 309–365. An English trans. (with references to Padoa's method) is in Jean van Heijenoort, ed., *From Frege to Gödel: A Source Book in Mathematical Logic 1879–1931* (Cambridge, Mass., 1967), 118–123. Padoa's major work is "La logique déductive dans sa dernière phase de développement," in *Revue de métaphysique et de morale*, 19 (1911), 828–832; 20 (1912), 48–67, 207–231, also published separately, with a preface by G. Peano (Paris, 1912).

II. SECONDARY LITERATURE. There is no biography of Padoa. Some information on his life and work may be found in the obituaries in *Bollcttino dell'Unione matematica italiano*, 16 (1937), 248; and *Revue de métaphysique et de morale*, 45 (1938), Apr. supp., 32: and in F. G. Tricomi, "Matematici italiani del primo secolo dello stato unitario," in *Memorie della Accademia delle scienze di Torino*, 4th ser., no. 1 (1962), 81.

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*Dictionary of Scientific Biography*, ed. C. C. Gillispie. New York: Charles Scribner's Sons. 10 (1974): 327–328.

PARSEVAL DES CHÊNES, MARC-ANTOINE (b. Rosières-aux-Salines, France, 27 April 1755; d. Paris, France, 16 August 1836), mathematics.

Little is known of Parseval's life or work. He was a member of a distinguished French family and described himself as a squire; his marriage in 1795 to Ursule Guerillot soon ended in divorce. An ardent royalist, he was imprisoned in 1792 and later fled the country when Napoleon ordered his arrest for publishing poetry against the regime. He was nominated for election to the Paris Academy of Sciences in 1796, 1799, 1802, 1813, and 1828; but the closest he came to being elected was to place third to Lacroix in 1799.

Parseval's only publications seem to have been five memoirs presented to the Academy of Sciences. The second of these (dated 5 April 1799) contains the famous Parseval theorem, given here in his own notation:

If there are two series

$$A + Bf + Cf^2 + Ff^3 + \cdots = T$$

$$a + b\frac{1}{f} + c\frac{1}{f^2} + f\frac{1}{f^3} + \cdots = T'$$

as well as the respective sums $T$, $T'$, then we obtain the sum of the series

$$Aa + Bb + Cc + Ff + \cdots = V$$

by multiplying $T$ by $T'$ and, in the new function $T \times T'$, substituting

$$\cos u + \sqrt{-1}\ \sin u$$

for the variable $f$, which will yield the function $V'$. Then for $f$ substitute

$$\cos u - \sqrt{-1}\ \sin u$$

which will yield the new function $V''$. We then obtain

$$V = \frac{1}{u} \int \frac{V' + V''}{2}\, du,$$

$u$ being made equal to 180° after integrating.

According to Parseval, the theorem was suggested by a method of summing special cases of series of products, presented by Euler in his *Institutiones calculi differentialis* of 1755, He believed the theorem to be self-evident, suggesting that the reader multiply the two series and recall that $(\cos u + i \sin u)^m = \cos mu + i \sin mu$, and gave a simple example that would "confirm its validity." He noted that it could be used only if the imaginaries in $V'$ and $V''$ cancel one another, and he hoped to overcome this inconvenience. This hope was realized in a note appended to his next memoir (dated 5 July 1801), in which he gave a simplified version of the theorem. In modern notation the theorem states:

If, in the series $M = A + Bs + Cs^2 + \cdots$ and $m = a + bs + cs^2 + \cdots$, $s$ is replaced by $\cos u + i \sin u$, and the real and imaginary parts are separated so that

$$M = P + Qi$$

and

$$m = p + qi,$$

then

$$\frac{2}{\pi} \int_0^\pi Pp\, du = 2Aa + Bb + Cc + \cdots .$$

(There is an error in Parseval's statement: the 2 in the right-hand side of the last equation is missing.)

In his memoirs, which were not published until 1806, Parseval applied his theorem to the solution of certain differential equations suggested by Lagrange and d'Alembert. The theorem first appeared in print in 1800, in Lacroix's *Traité des différences et des séries* (p. 377). By 1810 Delambre, in his *Rapport historique sur les progrès des sciences mathématiques depuis 1789, et sur leur état actuel*, could report that Prony had given, and published, lectures at the École Polytechnique taking Parseval's procedure into account and that Poisson had used a method dependent on an equation of this type. Since then dozens of equations have been called Parseval equations, although some only remotely resemble the original. Although Parseval's method involves trigonometric series, he never tried to find a general expression for the series coefficients; and hence he did not contribute directly to the theory of Fourier series. It should he noted that although Parseval viewed his theorem as a formula for summing infinite series, it was taken up at the end of the century as defining properties in more abstract treatments of analysis.

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HUBERT C. KENNEDY

Giuseppe Peano (1858–1932)

*Dictionary of Scientific Biography*, ed. C. C. Gillispie. New York: Charles Scribner’s Sons. 10 (1974): 441–444.

PEANO, GIUSEPPE (b. Spinetta, near Cuneo, Italy, 27 August 1858; d. Turin, Italy, 20 April 1932), *mathematics, logic*.

Giuseppe Peano was the second of the five children of Bartolomeo Peano and Rosa Cavallo. His brother Michele was seven years older. There were two younger brothers, Francesco and Bartolomeo, and a sister, Rosa. Peano’s first home was the farm Tetto Galant, near the village of Spinetta, three miles from Cuneo. the capital of Cuneo province, in Piedmont. When Peano entered school, both he and his brother walked the distance to Cuneo each day. The family later moved to Cuneo so that the children would not have so far to walk. The older brother became a successful surveyor and remained in Cuneo. In 1974 Tetto Galant was still in the possession of the Peano family.

Peano’s maternal uncle, Michele Cavallo, a priest and lawyer, lived in Turin. On this uncle’s invitation Peano moved to Turin when he was twelve or thirteen. There he received private lessons (some from his uncle) and studied on his own, so that in 1873 he was able to pass the lower secondary examination of the Cavour School. He then attended the school as a regular pupil and in 1876 completed the upper secondary program. His performance won him a room-and-board scholarship at the Collegio delle Provincie, which was established to assist students from the provinces to attend the University of Turin.

Peano’s professors of mathematics at the University of Turin included Enrico D’Ovidio, Angelo Genocchi, Francesco Siacci. Giuseppe Basso, Francesco Faà di Bruno. and Giuseppe Erba. On 16 July 1880 he completed his final examination “with high honors.” For the academic year 1880–1881 he was assistant to D’Ovidio, From the fall of 1881 he was assistant and later substitute for Genocchi until the latter’s death in 1889. On 21 July 1887 Peano married Carola Crosio, whose father, Luigi Crosio (1835–1915), was a genre painter.

On 1 December 1890, after regular competition, Peano was named extraordinary professor of infinitesimal calculus at the University of Turin. He was promoted to ordinary professor in 1895. In 1886 he had been named professor at the military academy, which was close to the university. In 1901 he gave up his position at the military academy but retained his professorship at the university until his death in 1932, having transferred in 1931 to the chair of complementary mathematics. He was elected to a number of scientific societies, among them the Academy of Sciences of Turin, in which he played a very active role. He was also a knight of the Order of the Crown of Italy and of the Order of Saint Maurizio and Saint Lazzaro. Although he was not active politically, his views tended toward socialism; and he once invited a group of striking textile workers to a party at his home. During World War I he advocated a closer federation of the allied countries, to better prosecute the war and, after the peace, to form the nucleus of a world federation. Peano was a nonpracticing Roman Catholic.

Peano's father died in 1888; his mother, in 1910. Although he was rather frail as a child, Peano's health was generally good. His most serious illness was an attack of smallpox in August 1889. After having taught his regular class the previous afternoon, Peano died of a heart attack the morning of 20 April 1932. At his request the funeral was very simple, and he was buried in the Turin General Cemetery. Peano was survived by his wife (who died in Turin on 9 April 1940), his sister, and a brother. He had no children. In 1963 his remains were transferred to the family tomb in Spinetta.

Peano is perhaps most widely known as a pioneer of symbolic logic and a promoter of the axiomatic method, but he considered his work in analysis most important. In 1915 he printed a list of his publications, adding: "My works refer especially to infinitesimal calculus, and they have not been entirely useless, seeing that, in the judgment of competent persons, they contributed to the constitution of this science as we have it today." This "judgment of competent persons" refers in part to the *Encyklopädie der mathematischen Wissenschaften*, in which Alfred Pringsheim lists two of Peano's books among nineteen important calculus texts since the time of Euler and Cauchy. The first of these books was Peano's first major publication and is something of an oddity in the history of mathematics, since the title page gives the author as Angelo Genocchi, not Peano: *Angelo Genocchi, Calcolo differenziale e principii di calcolo integrale, publicato con aggiunte dal D.$^{r}$ Giuseppe Peano*. The origin of the book is that Bocca Brothers wished to publish a calculus text based on Genocchi's lectures. Genocchi did not wish to write such a text but gave Peano permission to do so. After its publication Genocchi, thinking Peano lacked regard for him, publicly disclaimed all credit for the book, for which Peano then assumed full responsibility.

Of the many notable things in this book, the *Encyklopädie der mathematischen Wissenschaften* cites theorems and remarks on limits of indeterminate expressions, pointing out errors in the better texts then in use; a generalization of the mean-value theorem for derivatives: a theorem on uniform continuity of functions of several variables; theorems on the existence and differentiability of implicit functions; an example of a function the partial derivatives of which do not commute; conditions for expressing a function of several variables with Taylor's formula; a counterexample to the current theory of minima; and rules for integrating rational functions when roots of the denominator are not known. The other text of Peano cited in the *Encyklopädie* was the two-volume *Lezioni di analisi infinitesimale* of 1893. This work contains fewer new results but is notable for its rigor and clarity of exposition.

Peano began publication in 1881 with articles on the theory of connectivity and of algebraic forms. They were along the lines of work done by D'Ovidio and Faà di Bruno. Peano's work in analysis began in 1883 with an article on the integrability of functions. The article of 1890 contains original notions of integrals and areas. Peano was the first to show that the first-order differential equation $y' = f(x,y)$ is solvable on the sole assumption that $f$ is continuous. His first proof dates from 1886, but its rigor leaves something to be desired. In 1890 this result was generalized to systems of differential equations using a different method of proof. This work is also notable for containing the first explicit statement of the axiom of choice. Peano rejected the axiom of choice as being outside the ordinary logic used in mathematical proofs. In the *Calcolo geometrico* of 1884 Peano had already given many counterexamples to commonly accepted notions in mathematics, but his most famous example was the space-filling curve that was published in 1890. This curve is given by continuous parametric functions and goes through every point in a square as the parameter ranges over some interval. Some of Peano's work in analysis was quite original, and he has not always been given credit for his priority; but much of his publication was designed to clarify and to make rigorous the current definitions and theories. In this regard we may mention his clarification of the notion of area of a surface (1882, independently discovered by H. A. Schwarz); his work with Wronskians, Jacobians, and other special determinants, and with Taylor's formula; and his generalizations of quadrature formulas.

Peano's work in logic and in the foundations of mathematics may be considered together, although he never subscribed to Bertrand Russell's reduction of mathematics to logic. Peano's first publication in logic was a twenty-page preliminary section on the operations of deductive logic in *Calcolo geometrico secondo 1'Ausdehnungslehre di H. Grassmann* (1888). This section, which has almost no connection with the rest of the text, is a synthesis of, and improvement on, some of the work of Boole, Schröder, Peirce, and McColl. The following year, with the publication of *Arithmetices principia, nova methodo exposita*. Peano not only improved his logical symbolism but also used his new method to achieve important new results in mathematics; this short booklet contains Peano's first statement of his famous postulates for the natural numbers, perhaps the best known of all his creations. His research was done independently of the work of Dedekind, who the previous year had published an analysis of the natural numbers, which was essentially that of Peano but without the clarity of Peano. (This was the only work Peano wrote in Latin.) *Arithmetices principia* made important innovations in logical notation, such as $\epsilon$ for set membership and a new notation for universal quantification. Indeed, much of Peano's notation found its way, either directly or in a somewhat modified form, into mid-twentieth-century logic.

In the 1890s he continued his development of logic, and he presented an exposition of his system to the First International Congress of Mathematicians (Zurich, 1897). At the Paris Philosophical Congress of 1900, Peano and his collaborators—Burali-Forti, Padoa, and Pieri—dominated the discussion. Bertrand Russell later wrote, "The Congress was a turning point in my intellectual life, because I there met Peano."

In 1891 Peano founded the journal *Rivista di matematica*, which continued publication until 1906. In the journal were published the results of his research and that of his followers, in logic and the foundations of mathematics. In 1892 he announced in the *Rivista* the *Formulario* project, which was to take much of his mathematical and editorial energies for the next sixteen years. He hoped that the result of this project would be the publication of a collection of all known theorems in the various branches of mathematics. The notations of his mathematical logic were to be used, and proofs of the theorems were to be given. There were five editions of the *Formulario*. The first appeared in 1895; the last was completed in 1908, and contained some 4,200 theorems. But Peano was less interested in logic as a science per se than in logic as used in mathematics. (For this reason he called his system "mathematical logic.") Thus the last two editions of the *Formulario* introduce sections on logic only as it is needed in the proofs of mathematical theorems. The editions through 1901 do contain separate, well-organized sections on logic.

The postulates for the natural numbers received minor modifications after 1889 and assumed their definitive form in 1898. Peano was aware that the postulates do not characterize the natural numbers and, therefore, do not furnish a definition of "number." Nor did he use his mathematical logic for the reduction of mathematical concepts to logical concepts. Indeed, he denied the validity of such a reduction. In a letter to Felix Klein (19 September 1894) he wrote: "The purpose of mathematical logic is to analyze the ideas and reasoning that especially figure in the mathematical sciences." Peano was neither a logicist nor a formalist. He believed rather that mathematical ideas are ultimately derived from our experience of the material world.

In addition to his research in logic and arithmetic, Peano also applied the axiomatic method to other fields, notably geometry, for which he gave several axiom systems. His first axiomatic treatment of elementary geometry appeared in 1889 and was extended in 1894. His work was based on that of Pasch but reduced the number of undefined terms from four to three: point and segment, for the geometry of position (1889), and motion, also necessary for metric geometry (1894). (This number was reduced to two by Pieri in 1899.)

The treatise *Applicazioni geometriche del calcolo infinitesimale* (1887) was based on a course Peano began teaching at the University of Turin in 1885 and contains the beginnings of his "geometrical calculus" (here still influenced by Bellavitis' method of equipollences), new forms of remainders in quadrature formulas, new definitions of length of an arc of a curve and of area of a surface, the notion of a figure tangent to a curve, a determination of the error term in Simpson's formula, and the notion of the limit of a variable figure. There is also a discussion of the measure of a point set, of additive functions of sets, and of integration applied to sets. Peano here generalized the notion of measure that he had introduced in 1883. Peano's popularization of the vectorial methods of H.

Grassmann—beginning with the publication in 1888 of the *Calcolo geometrico secondo l'Ausdehnungslehre di H. Grassmann*—was of more importance in geometry. Grassmann's own publications have been criticized for their abstruseness. Nothing could be clearer than Peano's presentation, and he gave great impetus to the Italian school of vector analysis.

Peano's interest in numerical calculation led him to give formulas for the error terms in many commonly used quadrature formulas and to develop a theory of "gradual operations," which gave a new method for the resolution of numerical equations. From 1901 until 1906 he also contributed to actuarial mathematics, when as a member of a state commission he was asked to review a pension fund.

Peano also wrote articles on rational mechanics (1895–1896). Several of these articles dealt with the motion of the earth's axis and had their origin in the famous "falling cat" experiment of the Paris Academy of Sciences in the session of 29 October 1894. This experiment raised the question: "Can the earth change its own orientation in space, using only internal actions as animals do?" Peano took the occasion to apply his geometrical calculus in order to show that, for example, the Gulf Stream alone was able to alter the orientation of the earth's axis. This topic was the occasion of a brief polemic with Volterra over both priority and substance.

By 1900 Peano was already interested in an international auxiliary language, especially for science. On 26 December 1908 he was elected president of the Akademi Internasional de Lingu Universal, a continuation of the Kadem Volapüka, which had been organized in 1887 by the Reverend Johann Martin Schleyer in order to promote Volapük, the artificial language first published by Schleyer in 1879. Under Peano's guidance the Academy was transformed into a free discussion association, symbolized by the change of its name to Academia pro Interlingua in 1910. (The term "interlingua" was understood to represent the emerging language of the future.) Peano remained president of the Academia until his death. During these years Peano's role as interlinguist eclipsed his role as professor of mathematics.

Peano's mathematical logic and his ideography for mathematics were his response to Leibniz' dream of a "universal characteristic," whereas Interlingua was to be the modern substitute for medieval Latin, that is, an international language for scholars, especially scientists. Peano's proposal for an "interlingua" was *latino sine flexione* ("Latin without grammar"), which he published in 1903. He believed that there already existed an international scientific vocabulary, principally of Latin origin; and he tried to select the form of each word which would be most readily recognized by those whose native language was either English or a Romance language. He thought that the best grammar was no grammar, and he demonstrated how easily grammatical structure may be eliminated. His research led him to two areas: one was the algebra of grammar, and the other was philology. The latter preoccupation resulted most notably in *Vocabulario commune ad latino-italiano-français-english-deutsch* (1915), a greatly expanded version of an earlier publication (1909). This second edition contains some 14,000 entries and gives for each the form to be adopted in Interlingua, the classic Latin form, and its version in Italian, French, English, and German (and sometimes in other languages), with indications of synonyms, derivatives, and other items of information.

In his early years Peano was an inspiring teacher; but with the publication of the various editions of the *Formulario*, he adopted it as his text, and his lectures suffered from an excess of formalism. Because of objections to this method of teaching, he resigned from the military academy in 1901 and a few years later stopped lecturing at the Polytechnic. His interest in pedagogy was strong, and his influence was positive. He was active in the Mathesis Society of school teachers of mathematics (founded in 1895); and in 1914 he organized a series of conferences for secondary teachers of mathematics in Turin, which continued through 1919. Peano constantly sought to promote clarity, rigor, and simplicity in the teaching of mathematics. "Mathematical rigor," he wrote, "is very simple. It consists in affirming true statements and in not affirming what we know is not true. It does not consist in affirming every truth possible."

As historian of mathematics Peano contributed many precise indications of origins of mathematical terms and identified the first appearance of certain symbols and theorems. In his teaching of mathematics he recommended the study of original sources, and he always tried to see in his own work a continuation of the ideas of Leibniz, Newton, and others.

The influence of Peano on his contemporaries was great, most notably in the instance of Bertrand Russell. There was also a school of Peano: the collaborators on the *Formu-*

*lario* project and others who were proud to call themselves his disciples. Pieri, for example, had great success with the axiomatic method, Burali-Forti applied Peano's mathematical logic, and Burali-Forti and Marcolongo developed Peano's geometrical calculus into a form of vector analysis. A largely different group was attracted to Peano after his shift of interest to the promotion of an international auxiliary language. This group was even more devoted; and those such as Ugo Cassina, who shared both the mathematical and philological interests of Peano, felt the closest of all.

It has been said that the apostle in Peano impeded the work of the mathematician. This is no doubt true, especially of his later years; but there can be no question of his very real influence on the development of mathematics. He contributed in great measure to the popularity of the axiomatic method, and his discovery of the space-filling curve must be considered remarkable. While many of his notions, such as area and integral, were "in the air," his originality is undeniable. He was not an imposing person, and his gruff voice with its high degree of lallation could hardly have been attractive; but his gentle personality commanded respect, and his keen intellect inspired disciples. Much of Peano's mathematics is now of historical interest; but his summons to clarity and rigor in mathematics and its teaching continues to be relevant, and few have expressed this call more forcefully.

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*Dictionary of Scientific Biography*, ed. C. C. Gillispie. New York: Charles Scribner's Sons. 10 (1974): 605–606.

PIERI, MARIO (b. Lucca, Italy, 22 June 1860; d. Sant' Andrea di Compito (Lucca), Italy, 1 March 1913), *projective geometry, foundations of geometry*.

Pieri's father, Pellegrino Pieri, was a lawyer; his mother was Erminia Luporini. He began his university studies in 1880 at Bologna, where Salvatore Pincherle was among the first to recognize his talent; but he obtained a scholarship to the Scuola Normale Superiore of Pisa in November 1881 and completed his university studies there, receiving his degree on 27 June 1884. After teaching briefly at the technical secondary school in Pisa he became professor of projective geometry at the military academy in Turin and also, in 1888, assistant in projective geometry at the University of Turin, holding both posts until 1900. He became *libero docente* at the university in 1891 and for several years taught an elective course in projective geometry there.

On 30 January 1900, following a competition, he was named extraordinary professor of projective and descriptive geometry at the University of Catania. In 1908 he transferred to Parma, where in the winter of 1911 he began to complain of fatigue. His fatal illness, cancer, was diagnosed a few months later.

For ten years following his first publication in 1884, Pieri worked primarily in projective geometry. From 1895 he studied the foundations of mathematics, especially the axiomatic treatment of geometry. Pieri had made a thorough study of Christian von Staudt's geometry of position, but he was also influenced by his colleagues at the military academy and the university, Giuseppe Peano and Cesare Burali-Forti. He learned symbolic logic from the latter, and Peano's axiom systems for arithmetic and ordinary geometry furnished models for Pieri's axiomatic study of projective geometry.

In 1895 Pieri constructed ordinary projective geometry on three undefined terms: point, line, and segment. The same undefined terms were used in 1896 in an axiom system for the projective geometry of hyperspaces, and in 1897 he showed that all of the geometry of position can be based on only two undefined terms: projective point and the join of two projective points. In the memoir "I principii della geometria di posizione composti in un sistema logico-deduttivo" (1898) Pieri combined the results reached thus far into a more organic whole. Here the same two undefined terms were used to construct projective geometry as a logical-deductive system based on nineteen sequentially independent axioms—each independent of the preceding ones—which are introduced one by one as they are needed in the development, thus allowing the reader to determine on which axioms a given theorem depends. Of this paper Bertrand Russell wrote: “This is, in my opinion, the best work on the present subject” (*Principles of Mathematics*, 2nd ed. [New York, 1964], 382). a judgment that Peano echoed in his report in 1903 to the judging committee for the Lobachevsky Award of the Société Physico-Mathématique de Kasan. (Pieri received honorable mention, the prize going to David Hilbert.)

In their axiom systems for ordinary geometry, Pasch had used four undefined terms, and Peano three. With Pieri’s memoir of 1899, “Della geometria elementare come sistema ipotetico-deduttivo,” the number was reduced to two—point and motion—the latter understood as the transformation of one point into another. Pieri continued to apply the axiomatic method to the study of geometry, and in several subsequent publications he investigated the possibility of using different sets of undefined terms to construct various geometries. In “Nuovi principii di geometria proiettiva complessa” (1905) he gave the first axiom system for complex projective geometry that is not constructed on real projective geometry.

Two brief notes published in 1906–1907 on the foundations of arithmetic are notable. In “Sur la compatibilité des axiomes de l’arithmétique” he gave an interpretation of the notion of whole number in the context of the logic of classes; and in “Sopra gli assiomi aritmetici” he selected as primitive notions “number” and “successor of a number,” and characterized them with a system of axioms that from a logical point of view simplified Peano’s theory. In 1911 Pieri may have been on the point of beginning a new phase of his scientific activity. He was then attracted by the vectorial calculus of Burali-Forti and Roberto Marcolongo, but he left only three notes on this subject.

Pieri became one of the strongest admirers of symbolic logic: and although most of his works are published in more ordinary mathematical language, the statements of colleagues and his own statements show that Pieri considered the use of Peano’s symbolism of the greatest help not only in obtaining rigor but also in deriving new results.

Pieri was among the first to promote the idea of geometry as a hypothetical-deductive system. His address at the First International Congress of Philosophy in 1900 had the highly significant title "Sur la géométrie envisagée comme un système purement logique." Bertrand Russell wrote in 1903: "The true founder of non-quantitative Geometry is von Staudt. . . . But there remained one further step, before projective Geometry could be considered complete, and this step was taken by Pieri. . . . Thus at last the long process by which projective Geometry has purified itself from every metrical taint is completed" (*Principles of Mathematics*, 2nd ed. [New York, 1964]. 421).

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HUBERT C. KENNEDY

Emil Leon Post (1897–1954)

*Dictionary of Scientific Biography*, ed. C. C. Gillispie. New York: Charles Scribner's Sons. 11 (1975): 106–108.

POST, EMIL LEON (b. Augustów, Poland, 11 February 1897; d. northern New York, 21 April 1954), *mathematics, logic*.

Post was the son of Arnold J. and Pearl D. Post. In May 1904 he arrived in America, where his father and his uncle, J. L. Post, were in the fur and clothing business in New York. As a child Post's first love was astronomy, but the loss of his left arm when he was about twelve ruled that out as a profession. He early showed mathematical ability, however; and his important paper on generalized differentiation, although not published until 1930, was essentially completed by the time he received the B.S. from the College of the City of New York in 1917. Post was a graduate student, and later lecturer, in mathematics at Columbia University from 1917 to 1920, receiving the A.M. in 1918 and the Ph.D. in 1920. After receiving the doctorate, Post was a Proctor fellow at Princeton University for a year and then returned to Columbia as instructor, but after a year he suffered the first of the recurrent periods of illness that partially curtailed his scientific work. In the spring of 1924 he taught at Cornell University but again became ill. He resumed his teaching in the New York City high schools in 1927. Appointed to City College in 1932, he stayed there only briefly, returning in 1935 to remain for nineteen years. Post's family was Jewish; while not orthodox in his adult years, he was a religious man and proud of his heritage. He married Gertrude Singer on 25 December 1929 and they had one daughter.

Post was a member of the American Mathematical Society from 1918 and a member of the Association for Symbolic Logic from its founding in 1936. His extra-scientific interests included sketching, poetry, and stargazing.

Post was the first to obtain decisive results in finitistic metamathematics when, in his Ph.D. dissertation of 1920 (published in 1921), he proved the consistency as well as the completeness of the prepositional calculus as developed in Whitehead and Russell's *Principia Mathematica*. This marked the beginning, in important respects, of modern proof theory. In this paper Post systematically applied the truth-table method, which had been introduced into symbolic logic by C. S. Peirce and Ernst Schröder. (Post gave credit for his method to Cassius J. Keyser when he dedicated his *Two-Valued Iterative Systems* to Keyser, “in one of whose pedagogical devices the author belatedly recognizes the true source of his truth-table method.”) From this paper came general notions of completeness and consistency: A system is said to be complete in Post’s sense if every well-formed formula becomes provable if we add to the axioms any well-formed formula that is not provable. A system is said to be consistent in Post’s sense if no well-formed formula consisting of only a prepositional variable is provable. In this paper Post also showed how to set up multivalued systems of prepositional logic and introduced multivalued truth tables in analyzing them. Jan Łukasiewicz was studying three-valued logic at the same time; but while his interest was philosophical, Post’s was mathematical. Post compared these multivalued systems to geometry, noting that they seem “to have the same relation to ordinary logic that geometry in a space of an arbitrary number of dimensions has to the geometry of Euclid.”

Post began a scientific diary in 1916 and so was able to show, in a paper written in 1941 (but rejected by a mathematics journal and not published until 1965), that he had attained results in the 1920s similar to those published in the 1930s by Kurt Gödel, Alonzo Church, and A. M. Turing. In particular, he had planned in 1925 to show through a special analysis that *Principia Mathematica* was inadequate but later decided in favor of working for a more general result, of which the incompleteness of the logic of *Principia* would be a corollary. This plan, as Post remarked, “did not count on the appearance of a Gödel!”

If Post’s interest in 1920 in multivalued logics was mathematical, he also wrote in his diary about that time: “I study Mathematics as a product of the human mind and not as absolute.” Indeed, he showed an increasing interest in the creative process and noted in 1941 that “perhaps the greatest service the present account could render would stem from its stressing of its final conclusion that mathematical thinking is, and must be, essentially creative.”‘ But this is a creativity with limitations, and he saw symbolic logic as “the indisputable means for revealing and developing these limitations.”

On the occasion of Post’s death in 1954, W. V. Quine wrote:

> Modern proof theory, and likewise the modern theory of machine computation, hinge on the concept of a recursive function. This important number-theoretic concept, a

> precise mathematical substitute for the vague idea of “effectiveness” or “computability,” was discovered independently and in very disparate but equivalent forms by four mathematicians, and one of these was Post. Subsequent work by Post was instrumental to the further progress of the theory of recursive functions.

If other mathematicians failed to recognize the power of this theory, it was forcefully shown to them in 1947, when Post demonstrated the recursive unsolvability of the word problem for semigroups, thus solving a problem proposed by A. Thue in 1914. (An equivalent result had been obtained by A. A. Markov.) When reminded of his earlier statement, Quine in 1972 confirmed his opinion, adding: “The theory of recursive functions, of which Post was a co-founder, is now nearly twice as old as it was when I wrote that letter. What a fertile field it has proved to be!”

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II. SECONDARY LITERATURE. There is an obituary of E. L. Post in *The Campus* (City College), 27 April 1954. Part of Post’s work was carried to a conclusion by S. V. Yablonsky and his students. See S. V. Yablonsky, G. P. Gavrilov, V. B. Kudryavtsev, *Funktsii algebry logiki i klassy Posta* (Moscow, 1966); translated into German as *Boolesche Funktionen und Postsche Klassen* (Brunswick, 1970).

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Giovanni Vailati (1863–1909)

*Dictionary of Scientific Biography*, ed. C. C. Gillispie. New York: Charles Scribner's Sons. 13 (1976): 550–551.

VAILATI, GIOVANNI (b. Crema, Italy, 24 April 1863; d. Rome Italy, 14 May 1909), *logic, philosophy of science, history of science*.

Vailati's parents were Vincenzo Vailati and Teresa Albergoni. After attending boarding schools in Monza and Lodi, he enrolled in the University of Turin in 1880, graduating in engineering in 1884 and in mathematics in 1888. Then followed a period of independent study in which he especially studied languages (his writings show a proficiency in Greek, Latin, English, French, German, and Spanish); this was interrupted by the offer of an assistantship at the University of Turin by his former teacher Giuseppe Peano, professor of infinitesimal calculus. Vailati was Peano's assistant from 1892 to 1895, when he became an assistant in projective geometry and later honorary assistant to Volterra, In 1899 he requested a secondary school appointment and was at first sent to Syracuse, transferring to Bari in 1900, to Como in 1901, and to Florence in 1904.

Vailati came of a Catholic family but lost his faith during his early university years. Throughout his life he had affectionate and devoted friends; he never married. His premature death was attributed to heart trouble, complicated by pulmonitis.

Vailaii's first ten publications, dealing principally with mathematical logic, were published in the *Rivista di matematica*, founded by Peano in 1891. He also collaborated, especially with historical notes, in the *Formulario* project announced by Peano in 1892, Vailati gained international recognition with the publication of three essays in the history and methodology of science, originally given as introductory lectures to his course in the history of mechanics at the University of Turin (1896–1898).

Vailati was always concerned with tracing ideas back to their origins, and his intimate knowledge of Greek and Latin was invaluable. (In the analytical index to the *Scritti*, "Aristotle" has twice the space of any other entry.) His work in this area will perhaps be his most lasting contribution.

Vailati received most attention during his lifetime as the leading Italian exponent of pragmatism. After his transfer to Florence in 1904 he collaborated. along with his friend and disciple Mario Calderoni, in the publication of the journal *Leonardo*, founded the year before by G. Papini and G. Prezzolini. His philosophical position was closer to that of Charles Sanders Peirce than to the more popular William James, but it remained distinct, individual, and original.

Vailati's wide range of interest included, at various periods, psychic research, economics, and political science (in which he took socialism seriously, but opposed Marx's theory of value). In all of these areas his acute critical sense allowed him, as was often said, "to succeed in saying in a few words what others had succeeded in not saying in many volumes." When the occasion seemed to call for it, he did not hesitate to criticize sharply the opinions of even eminent scientists (for example, he criticized Poincaré's views on mathematical logic).

Finally, Vailati's pedagogical activities must be noted, in recognition of which he was appointed a member of the commission for the reform of the secondary schools. For the work of the commission he established his residence in Rome in 1906, dividing his time between there and Florence, but in 1908 he voluntarily returned to teaching in Florence.

After his death, Vailati's reputation quickly suffered an eclipse; this was partly the result of the form in which his writings appeared. He never published a book-length monograph. Indeed, many of his original ideas appeared in critical reviews, which occupy, by page count, approximately 43 percent of the *Scritti*. After 1950 there was a revival of interest in his work, centering mainly on his philosophical views, but hindered by the general unavailability of his writings. Vailati also carried on a wide correspondence, which is mostly unpublished. Projects announced in 1958 for the publication of his correspondence and a new edition of his writings were not carried out.

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